

Textbook and Course Materials for 21-127 “Concepts of Mathematics”

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Abstract

Concepts of Mathematics (21-127 at CMU) is a course designed to introduce students to the world of abstract mathematics, guiding them from more calculation-based math (that one learns in high school) to higher mathematics, which focuses more on abstract thinking, problem solving, and writing **proofs**. This transition tends to be a shock: new notation, terminology, and expectations (particularly the requirement of **writing** mathematical thoughts formally) give students much trouble. Many standard texts for this course are found to be too dense, dry and formal by the students. Ultimately, this means these texts are unhelpful for student learning, despite their intentions. For this project, I have written a new textbook designed specifically for this course and the way it has been taught. My emphases—writing more informal prose, properly motivating all new material, and including a wider variety of exercises and questions for the reader—all reflect the goal of increasing the reader’s interest and potential for learning. The selection of material, examples, explanations, and problems has been supported by in-classroom experience, and their presentation has been influenced greatly by educational research. I have also included accompanying class notes, meant to supplement the teaching of this course, as well as assignments and exams, and their corresponding solutions and grading rubrics. Ultimately, this project constitutes a standalone packet for future instructors of this course and similar ones.

Acknowledgements

This work was completed jointly with my advisors, Profs. John Mackey and Jack Schaeffer, with additional guidance from my other thesis committee members, Prof. William Hrusa and Dr. Hilary E.Y. Schuldt of Carnegie Mellon's Eberly Center for Teaching Excellence & Educational Innovation.

Prof. Mackey oversaw the genesis of this project and much of the initial writing. He provided invaluable insight from his years of teaching this course, Concepts of Mathematics, and helped to give this project a sense of direction and establish purposeful goals. Many decisions about the inclusion and sequencing of material were influenced by his observations and suggestions, so I have cited him as a second author on the textbook's title page. He has also been an incredibly helpful resource for information and tips about teaching and handling a large lecture-style course, which was a very new situation for me. Whenever I had a question or tough decision to make, I went to John for advice, and he was always right. I thank him, as well, for letting me use some of my materials (including homework problems, for example) in the course when I served as his teaching assistant.

Prof. Schaeffer served as a fellow instructor and was the first person other than me to also use this textbook as the sole source material for the teaching of the course. I thank him, first of all, for trusting my project enough to use it concurrently. He was also helpful throughout that semester, providing many suggestions about the material, how to present it, and identifying errors. Several exercises that appear in the book were created and/or inspired by him, as well. His patient analysis of my writing, insightful suggestions, and general enthusiasm for the project have been very helpful.

Prof. Hrusa has always been supportive and offers wonderful advice. He deserves credit for helping with many of the logistical issues surrounding my teaching and the development of this project. In particular, I thank him and the entire Mathematical Sciences department for providing me the opportunity to serve as an instructor while still a graduate student. This is not common for the department to do, and it was an incredible learning opportunity and experience for me.

Dr. Schuldt provided an important perspective of this project from the view of an education expert. She forced me to think more carefully about how my writing addresses particular aspects of student learning, and provided many specific suggestions about how and where to do so, and why. Conversing with her kept me thinking about some of the broader goals of the project that might otherwise have been lost in the details.

I made use of a few technological elements while working on this project and teaching this course. In particular, I thank Adam Blank for developing the website AnnotateMyPDF and allowing me to test it out and use it. It proved to be not only a way to share my materials securely with my students, but also an extremely helpful way for them to provide feedback to me. Countless edits and

suggestions were received via this interface. Without it, I'm sure there would be three dozen extra typos in this project yet to be found. Adam is also opinionated and enthusiastic about education, and numerous conversations with him have provided me with many new ideas and perspectives.

I thank my fellow math graduate students and other friends who were always willing to “talk shop” and look over my materials. This includes Lisa, Spencer, Paul, Will B., Will G., Deepak, Marla, Emily, Brian, Chris, Rumi, Jason, Greggo, Michael, and many others. I also thank the TAs and graders who I've worked with in the past, and especially those who worked under me in the Fall 2012 semester.

I thank my parents and sister for being supportive and listening patiently (most of the time). While my own esteem might have wavered up and down, they have always been a positive reminder and a source of pride.

Most of all, I thank all of the students I've had over the years, both in the Concepts of Mathematics course and numerous others. This project is meaningless without them and future generations of students looking to learn more about the world of mathematics (even if they're being forced to do so). A not insignificant portion of my students have helped with the editing process, as well, by pointing out typos and other mistakes, suggesting material to be included/omitted, and pointing out particular examples and explanations that were helpful or redundant (as the case may be). They have made my work enjoyable and have continually reminded me of why I started this project in the first place.

Outline of Project

In this document, you will find essentially two things:

(1) a textbook, and (2) a set of course materials.

The textbook appears first. It was written to be used in this course and has been developed concurrently with several iterations of teaching the course, both during summer sessions and the academic year. It can serve as a standalone text for any iteration of this course, but I also believe it can be adapted for use in similar courses at other universities. Furthermore, I believe the “informal” nature of the writing style might make this text useful in a slower-paced course for non-majors, perhaps as a two-semester sequence. I also think the text can stand alone and be helpful to a motivated reader for self-study, as well as to a former student as a reference.

The course materials appear afterwards, and are all taken from the materials I used in the teaching of this course during the Fall 2012 semester. They appear in this particular order:

- *Syllabus*

This outlines the content of the course and many of the administrative details required to teach the course. This provides some context for the project and, in particular, describes the “Prep Questions” which are mentioned below in the *Exams* section.

- *Lecture notes*
These are the notes I used and shared with students. They might serve to indicate to a future instructor not only what particular parts of the textbook merit in-class discussion (and the corresponding depth of presentation) but also what sort of administrative (and otherwise tangential) details might come up during this course.
- *Recitation sheets*
These are the sheets I sent to my TAs to guide their preparation for recitations (in-class sessions with the TAs outside of lecture), and were distributed by them to the students. They are a nice written record of material for future students to have, but also serve as a source of extra examples and exercises for an instructor to potentially use.
- *Homeworks (with solutions and rubrics)*
These are the ten homework assignments I gave. They might serve as a guide for which exercises in the textbook are particularly instructive to formally assign. I have also included solutions (for both instructor and student reference) as well as some notes about how to grade these problems. These rubrics indicate what I believe to be important components of these problems and, by extension, what I think are essential skills for a student to take away from this book/course.
- *Exams (with review sheets, and exam solutions and rubrics)*
These are three in-class exams I gave, as well as the final exam. They might serve as a guide for an instructor about what students can reasonably achieve in a 50-minute, in-class exam. They were also carefully designed to not only address the breadth of the course material, but also particular skills the students should be developing. That is to say, they address students' conceptual, procedural, and applicable knowledge, not just the amount of information they have internalized. These also include corresponding solutions and rubrics, for the instructor's use.
One method I used to attempt to broaden the type and number of questions I could pose on an exam was to use review sheets with "Prep Questions", which are included here. As described in the syllabus, some of these questions would appear verbatim on the exams. Many of the questions contained therein are also exercises in the textbook, and can be helpful for both an instructor and a student looking for a list of problems to test the array of skills and content they should be internalizing and applying.

After this set of materials, I have included a list of references. These are books and articles and websites that I consulted and either cited directly in the textbook or else used as an inspiration for style, content, and exercises. Some of these books are other standard texts for a course like Concepts of Mathematics, and I believe it would behoove a reader to pick some of those up from a library/store and compare them to the textbook and materials I have shared here. I believe this will make the goals of my project much clearer and, hopefully, demonstrate how I have achieved those goals.

Thank you for reading! Peace and Math be with you.